

Introduction to the Special Issue on Contemporary Bayesian Prediction

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Prediction has always had a base level of popularity. Who doesn't want an infinitesimal peek into the future? In recent years, and more important for the present issue, we have seen ever greater interest in prediction in statistics. This has been driven by model uncertainty, data complexity, and the success of models that seek good prediction first, leaving physical modeling as a later task.

Indeed, the complexity of problems confronting the statistician has grown massively over the last few decades. While conventional theory as elucidated by classic textbooks such as [6] and [7] and their successors suffices for relatively traditional statistical problems, these are a decreasing fraction of the problems confronting the analyst today. Accordingly, a new paradigm must be invented.

The earliest treatment of prediction as a foundation for practical statistical inference seems to be [1]. However, [3] predates this and treats prediction abstractly as a Bayesian problem introducing concepts such as coherence and agreement among subjective Bayesians. Although [3] does not argue explicitly that prediction should be the foundation for inference, he uses a predictive framework to define a paradigm problem of statistics. In contrast, asserting that “prediction was the earliest and most prevalent form of statistical inference,” the text [5] bluntly states that its “principal intent is to revive the primary purpose of statistical endeavor, namely inferring about realizable values not observed based on values that were observed.”

Treating prediction as the “primary purpose of statistical endeavor” meshes smoothly with the dominant scientific paradigm: falsification of theories by comparing their predictions with empirical measurements as articulated in [8]. For falsification purposes, we do not need to distinguish between a physical modeling approach—however much it is desired—and purely predictive approaches that usually have a wider range of models and hence often achieve lower predictive error.

Aside from the desire to make and test predictions, what can we say in general about statistical techniques?

First, for methodology, the paradigm data set is no longer independent and identical outcomes conditional on

a random parameter. Instead, paradigmatically, we should think about large, streaming, possibly multitype, often observational data sets.

Second, we cannot rely on physical models the way we did in the past. The problem is that the complexity of the data generating model—assuming it exists—is so high relative to the information in the data, that modeling is prone to severe mis-specification. The complexity may mean that there is nothing stable enough to permit estimation or testing let alone modeling. Concisely put, all models are guilty of bias until proven innocent (even those that are false but useful).

Third, predictions should be conditional on past data and the default is to condition on all the available information. Conditioning is the main principled statistical technique for transferring information from one random variable to another. That is, once a data distribution has been assigned consistently, we can use it or its conditionals for the prediction of observables. There may be other techniques besides conditioning for prediction, but their adequacy comes down to how well they approximate a conditioning argument. We concede there may be other principled ways to transfer information from one variable to another but they are not commonly known.

Fourth, we think the authors represented here would accept as a “folk theorem” the view that any good statistical procedure is Bayesian or nearly so—even if different people have nontrivially different notions of what it means to be Bayesian. Indeed, a range of views is implicit (and sometimes explicit!) in the papers in this issue. Lack of consensus notwithstanding, one way this folk theorem can be formalized is in decision-theoretic terms: Bayes procedures generally form a complete class. Thus, for a given data distribution, the best action is Bayes or “ ϵ -close” in risk to a Bayes procedure; see [9], Chapter 8 for a good review and [2]. These results are for estimation, but it is no stretch to assume they can be adapted to prediction. This does not mean one should only use Bayesian solutions, but it does mean that there always is a Bayesian solution—if hard to find and use—that is essentially optimal.

Aside from the centrality of Bayesian prediction to contemporary Statistics as a field this issue has three general points to make:

1. Apart from bias, model-mis-specification and other statistical sins, the more data dependent a predictor is,

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that is, the less it depends on incompletely validated mathematical quantities, the better it is, especially for complex data types.

2. Prediction is improved when we comprehensively examine and critique the components that go into a predictor. This includes calibration as well as other mathematical features; see [4].
3. In addition to conditioning, we must have a mechanism for updating or, better, rechoosing the components of a predictor in response to performance.

This issue proceeds as follows. After a general introduction to the current state of Bayesian prediction by Clarke and Yao, the academic papers in this issue fall naturally into two groups.

First, we have three foundational papers.

- Patrizia Berti, Emanuela Dreassi, Fabrizio Leisen, Luca Pratelli, Pietro Rigo: “A probabilistic view on predictive constructions for Bayesian learning”;
- Sandra Fortini, Sonia Petrone: “Exchangeability, prediction and predictive modeling in Bayesian statistics”;
- Blake Moya, Stephen Walker: “Martingale posterior distributions for time series models.”

Second, we have three methodological papers.

- Sara Wade, Vanda Inacio: “Bayesian dependent mixture models: A predictive comparison and survey”;
- Matthew C. Johnson, Mike West: “Bayesian Predictive Synthesis with Outcome-Dependent Pools”;
- Yann McLatchie, Sölvi Rögnvaldsson, Frank Weber, Aki Vehtari: “Advances in projection predictive inference.”

Finally, yet essential, is an interview of Phil Dawid, one of the most influential thinkers in Bayesian predictive statistics. We hope all readers will enjoy the perspective borne of a lifetime of activity in one of the most important intellectual developments from the last few decades.

- Vladimir Vovk, Glenn Shafer: “A Conversation with A. Philip Dawid.”

In conclusion, no single issue can represent all the directions of contemporary Bayesian prediction. Part of the charm of Statistics is that it is continually rejuvenated by new data and the new questions they raise. However, we hope that, taken together, the selection of topics here gives a sense of the breadth and novelty being addressed within the Bayesian predictive context and serves as an indicator of future directions and achievements in this important and fast-moving field.

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